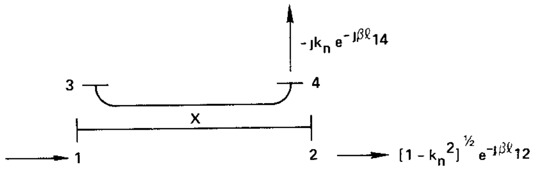

Fig. 6 Transfer function of ideal coupler C_n with k_n coupling coefficient

Using these equations, and noting that since $l_{bi} - l_{ai} = 0$ all the line length terms can be neglected, it can be shown that

$$q_2 = \frac{jk_2k_4}{k_1(1-k_2^2)^{1/2}}. \qquad \text{(A-8a)}$$

Similarly

$$q_3 = -\frac{k_3k_5 + jk_4(1-k_2^2)^{1/2}(1-k_3^2)^{1/2}}{k_1k_2(1-k_3^2)^{1/2}} \qquad \text{(A-9a)}$$

$$q_4 = \frac{k_5(1-k_3^2)^{1/2} - jk_3k_4(1-k_2^2)^{1/2}}{k_1k_2k_3}. \qquad \text{(A-10a)}$$

From the design values

$$k_1 = k_2 = k_3 = 0.708$$

and

$$k_4 = k_5 = 0.562.$$

Therefore

$$q_2 = j0.80 \qquad \text{(A-8b)}$$

$$q_3 = -1.12 - j0.80 \qquad \text{(A-9b)}$$

$$q_4 = 1.12 - j0.80. \qquad \text{(A-10b)}$$

Fig. 4 shows that the measured q-points were fairly close to these values.

Acknowledgment

The authors wish to acknowledge many helpful discussions with L. Susman of Sperry Research Center. The six-port junction could not have been constructed without the expertise and generous cooperation of T. Kozul of Baytron Co., Inc. T. Kirkland of SRC provided diligent assistance in performing the experiments. We would also like to thank M. Weidman of NBS for supplying the NBS Calibration Program.

References

[1] H. G. Oltman and H. A Leach, "A dual four-port for automatic network analysis," in *1981 IEEE MTT-S Internat. Symp. Dig.*, pp 69–72

[2] M. Weidman, Panel Session, "Millimeter wave automatic network analyzers," presented at 17th ARFTG Workshop, (Los Angeles, CA), June 18, 1981.

[3] Product Feature, Millimeter-wave network analyzer offers computer control, MSN, July 1981, p 98.

[4] G F. Engen, "The six-port reflectometer: An alternative network analyzer," *IEEE Trans Microwave Theory Tech.*, vol MTT-25, pp. 1075–1080, Dec 1977.

[5] H M Cronson and L. Susman, "A New calibration technique for automated broadband microwave measurements," in *Proc. 6th Euro. Microwave Conf.*, (Rome, Italy) Sept. 1976, p. 205.

[6] G. F. Engen, "Calibrating the six-port reflectometer by means of sliding terminations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 951–957, Dec. 1978.

[7] D Woods, "Analysis and calibration theory of the general six-port reflectometer employing four amplitude detectors," in *Proc. Inst. Elec. Eng.*, vol 126, Feb. 1979, pp. 221–228.

Analysis of the Shielded-Strip Transmission Line with an Anisotropic Medium

HISASHI SHIBATA, SHINYA MINAKAWA, AND RYUITI TERAKADO

***Abstract*—Analysis of the shielded stripline with an anisotropic medium is presented by means of an affine transformation and a conformal mapping technique. The capacitance is represented in the function $K'(k)/K(k)$ with a modified modulus k, which is obtained by multiplying d, the width between the ground planes in [1, table I], by α. The parameter α relates to the anisotropic medium. The exact distributions of equipotentials and lines of electric flux in the anisotropic medium are also presented.**

I. Introduction

A structure with homogenous isotropic medium in which the conductor is a strip of zero thickness located on the center inside two parallel infinite grounded planes has been exactly analyzed [1]–[7] by using a conformal mapping technique.

The object of this paper is to propose analytical formulas for the capacitance and field distributions of the structure with an anisotropic medium shown in Fig. 1(a). An affine transformation which was reported by Kusase and Terakado [8] is used to implement the present work. Application of the transformation leads us to analysis of the structure with a corresponding isotropic medium as shown in Fig. 1(b), and a conformal mapping technique is applied to the structure with the isotropic medium. Thus, the capacitance of the structure of Fig. 1(a) is represented in the function of complete elliptic integrals of the first kind, even though the stripline is composed of the anisotropic medium. The modulus which determines the elliptic integrals is merely obtained by multiplying d, the width between the ground planes in [1, table 1], by α. The parameter α is determined by the principal axes-relative dielectric constants of the anisotropic medium and the angle between the principal axes and the ground plane. In addition, we present an expansion for the capacitance in the series of $\exp(-b\pi/\alpha h)$, where b and h are dimensions of the structure. In Section III, we present the exact distributions of equipotentials and lines of electric flux in the structure shown in Fig. 1(a). The distributions will show that equipotential and flux lines do not perpendicularly intersect each other.

II. Affine and Conformal Transformations

Now consider the structure shown in Fig. 1(a). The two-dimensional space between two parallel infinite grounded planes is assumed to be filled with the anisotropic medium of the following permittivity tensor:

$$\bar{\epsilon} = \epsilon_0 \begin{pmatrix} \epsilon_{\parallel} & 0 \\ 0 & \epsilon_{\perp} \end{pmatrix} \qquad (1)$$

where $\epsilon_{\parallel}$, $\epsilon_{\perp}$, and ϵ_0 are the principal axes-relative dielectric constants of the anisotropic medium and the permittivity of vacuum, respectively. The tensor for the $x-y$ coordinates is obtained by rotating the principal axes with the angle θ as shown in

Manuscript received November 5, 1981; revised March 3, 1982.

H Shibata and S. Minakawa are with the Department of Electrical Engineering, Ibaraki Technical College, Katsuta, Ibaraki, 312, Japan.

R. Terakado is with the Department of Electrical Engineering, Faculty of Engineering, Ibaraki University, Hitachi, Ibaraki, 316, Japan

0018-9480/82/0800-1264$00.75 ©1982 IEEE

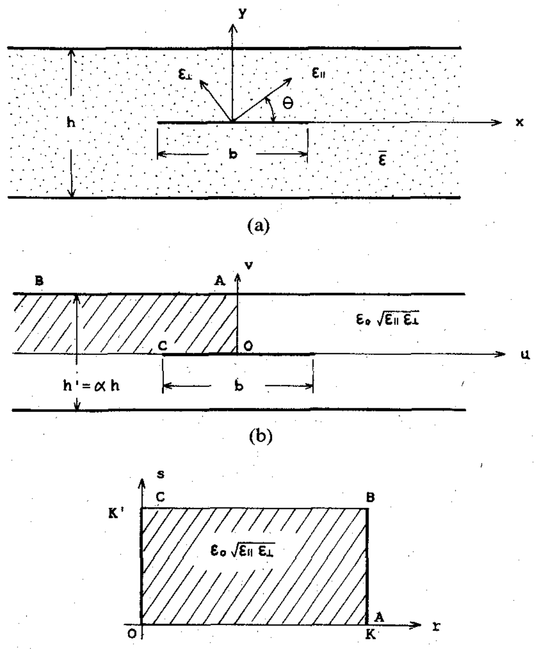

Fig. 1. Transformation from the shielded stripline with anisotropic medium to one with isotropic medium and conformal mapping. (a) z-plane. (b) w-plane. (c) t-plane.

Fig. 1(a), that is

$$\bar{\epsilon}(x,y)=\epsilon_0\begin{pmatrix}\epsilon_{\parallel}\cos^2\theta+\epsilon_{\perp}\sin^2\theta & (\epsilon_{\parallel}-\epsilon_{\perp})\sin\theta\cos\theta\\ (\epsilon_{\parallel}-\epsilon_{\perp})\sin\theta\cos\theta & \epsilon_{\perp}\cos^2\theta+\epsilon_{\parallel}\sin^2\theta\end{pmatrix}. \quad (2)$$

The structure with the medium of (2) is transformed to the structure shown in Fig. 1(b) with the isotropic medium of which the permittivity equals to $\epsilon_0\sqrt{\epsilon_{\parallel}\epsilon_{\perp}}$ by the following affine transformation [8]:

$$\begin{pmatrix}u\\v\end{pmatrix}=\begin{pmatrix}1 & \dfrac{(\epsilon_{\perp}/\epsilon_{\parallel}-1)\sin\theta\cos\theta}{\sin^2\theta+(\epsilon_{\perp}/\epsilon_{\parallel})\cos^2\theta}\\ 0 & \dfrac{\sqrt{\epsilon_{\perp}/\epsilon_{\parallel}}}{\sin^2\theta+(\epsilon_{\perp}/\epsilon_{\parallel})\cos^2\theta}\end{pmatrix}\cdot\begin{pmatrix}x\\y\end{pmatrix}. \quad (3)$$

Then the width between two parallel infinite grounded planes is converted into

$$h'=\alpha h \quad (4)$$

where

$$\alpha=\frac{\sqrt{\epsilon_{\perp}/\epsilon_{\parallel}}}{\sin^2\theta+(\epsilon_{\perp}/\epsilon_{\parallel})\cos^2\theta}. \quad (5)$$

But the width of the strip conductor located on the center of the structure is invariant. The transformation of (3) has the property that potential and energy are invariant. Therefore, the capacitances of both structures shown in Fig. 1(a) and (b) are equal. The analysis of the structure shown in Fig. 1(b) can be easily performed by the following conformal mapping function:

$$k^2\,\mathrm{sn}^2(t,k)=\frac{1-\cosh(2w\pi/\alpha h)}{\cosh(b\pi/\alpha h)-\cosh(2w\pi/\alpha h)} \quad (6)$$

where

$$k=\operatorname{sech}\left(\frac{b\pi}{2\alpha h}\right)\qquad k'=\tanh\left(\frac{b\pi}{2\alpha h}\right). \quad (7)$$

The function of (6) transforms the shaded quarter region of Fig. 1(b) (w-plane) into the rectangular region of Fig. 1(c) (t-plane). Therefore, the total capacitance of the structure shown in Fig. 1(a) can be obtained by

$$Ca=4\epsilon_0\sqrt{\epsilon_{\parallel}\epsilon_{\perp}}\,K'(k)/K(k) \quad (8)$$

where $K(k)$ and $K'(k)$ are complete elliptic integrals of the first kind.

III. Equipotential and Flux Lines

In Fig. 2, we show distributions of equipotentials and lines of electric flux in the structure of Fig. 1(a). The curves in z-plane that correspond to the lines with $r=$ const. and $s=$ const. in t-plane are concretely plotted by using the following formulas:

$$x=u-\frac{(\epsilon_{\perp}/\epsilon_{\parallel}-1)\sin\theta\cos\theta}{\sqrt{\epsilon_{\perp}/\epsilon_{\parallel}}}v,\qquad y=\frac{1}{\alpha}v \quad (9)$$

where

$$u=-\frac{\alpha h}{2\pi}\cdot\ln\frac{(1-2c)+\sqrt{2}\sqrt{c^2-d^2-c+\sqrt{(c^2+d^2)\{(1-c)^2+d^2\}}}}{\sqrt{(1-2c+2c^2+2d^2)-2\sqrt{(c^2+d^2)\{(1-c)^2+d^2\}}}}$$

$$v=\frac{\alpha h}{2\pi}\cdot\tan^{-1}\sqrt{\frac{c-c^2-d^2+\sqrt{(c^2+d^2)\{(1-c)^2+d^2\}}}{0.5-\left[c-c^2-d^2+\sqrt{(c^2+d^2)\{(1-c)^2+d^2\}}\right]}}$$

$$c=\frac{1}{2}\left(1-\cosh\frac{b\pi}{\alpha h}\right)\cdot\frac{(2k^2mn)^2-k^2(m^2-n^2)\{1-k^2(m^2-n^2)\}}{\{1-k^2(m^2-n^2)\}^2+(2k^2mn)^2}$$

$$d=-\left(1-\cosh\frac{b\pi}{\alpha h}\right)\cdot\frac{k^2mn}{\{1-k^2(m^2-n^2)\}^2+(2k^2mn)^2}$$

$$m=\frac{\mathrm{sn}(r,k)\,\mathrm{dn}(s,k')}{\mathrm{cn}^2(s,k')+k^2\mathrm{sn}^2(r,k)\,\mathrm{sn}^2(s,k')}$$

$$n=\frac{\mathrm{sn}(s,k')\,\mathrm{cn}(r,k)\,\mathrm{dn}(r,k)\,\mathrm{cn}(s,k')}{\mathrm{cn}^2(s,k')+k^2\mathrm{sn}^2(r,k)\,\mathrm{sn}^2(s,k')}.$$

Equation (9) was obtained by solving (3) and (6) for x and y, using the addition formula for the elliptic function $\mathrm{sn}(t,k)$. Fig. 2 shows the distributions with the parameters $\epsilon_{\perp}/\epsilon_{\parallel}=1/9$, $\theta=\pi/3$, and $b/h=0.75$. In this case, the capacitance of (8) is about $4.8\,\epsilon_0\sqrt{\epsilon_{\parallel}\epsilon_{\perp}}$; hence the numbers of ribbon between equipotential lines and the numbers of ribbon between lines of electric flux in Fig. 2 are chosen to be equal to 10 and 48,

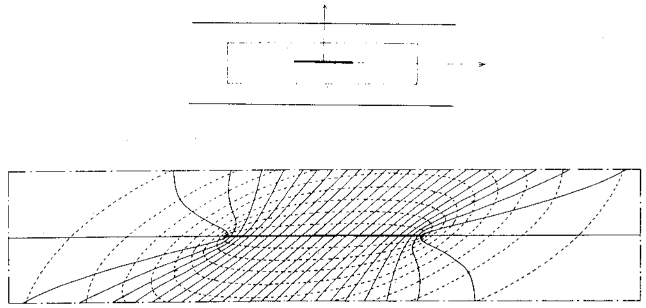

Fig. 2. Distributions of equipotentials and lines of electric flux of neighborhood of the strip conductor in the structure shown in Fig. 1(a) with $b/h = 0.75, \epsilon_\perp/\epsilon_\| = 1/9, \theta = \pi/6$. ------Equipotential line; ——line of electric flux.

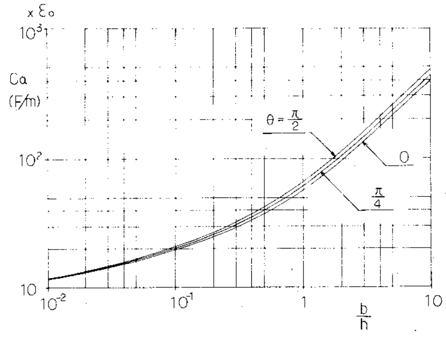

Fig. 3. The capacitance value versus b/h for the structure shown in Fig. 1(a) with sapphire substrate ($\epsilon_\| = 11.6, \epsilon_\perp = 9.4$).

respectively. Each capacitance of the quadrangles which are encircled by the lines adjacent to each other is about $\epsilon_0\sqrt{\epsilon_\|\epsilon_\|}$.

IV. Computation of Capacitance

The capacitance of the stripline with the anisotropic medium can be obtained by computing complete elliptic integrals as described in Section II. The evaluating process for the $K'(k)/K(k)$ is well known [9]. Also, the approximation for the K'/K or capacitance has been reported [1]–[3], [5]–[7].

In the computation of (8), Wheeler's iterative flow chart [7] is very useful for analysis and synthesis, because his flow chart gives swift convergence and very close approximation. However, it seems to be significant for analysis that the expansion for the capacitance is expanded in terms of the dimensions b, h of the structure and the parameter α, because the value of the capacitance can be easily evaluated without repetition. The expansion can be obtained by substituting (10) for the expansion of $K'(k)/K(k)$ shown in the Appendix and by employing (8)

$$\frac{1-k'}{k} = \exp\left(-\frac{b\pi}{2\alpha h}\right). \tag{10}$$

That is

$$Ca = \frac{4\epsilon_0\sqrt{\epsilon_\|\epsilon_\perp}}{\pi}\left\{\ln 4 + \frac{b\pi}{\alpha h} - \frac{1}{4}\exp\left(-2\frac{b\pi}{\alpha h}\right) - \frac{13}{128}\exp\left(-4\frac{b\pi}{\alpha h}\right) - \frac{23}{384}\exp\left(-6\frac{b\pi}{\alpha h}\right) - \frac{2701}{65536}\exp\left(-8\frac{b\pi}{\alpha h}\right) - \cdots\right\}$$
$$\left(\text{for } b/h \geqslant \alpha \ln\left(3+\sqrt{8}\right)/\pi\right). \tag{11}$$

Equation (11) gives the maximum error when $b/h = \alpha \ln(3+\sqrt{8})/\pi$. The value is 2.3×10^{-10} if we add up the first six terms of (11). The expansion of the capacitance for $b/h < \alpha \ln(3+\sqrt{8})/\pi$ can be obtained by substituting (12) into K'/K for $k^2 > 0.5$ which is presented in the Appendix

$$\frac{1-k}{k'} = \tanh\left(\frac{b\pi}{4\alpha h}\right). \tag{12}$$

But the expansion is not present in this paper.

As an example employing (11), the capacitance per unit length versus b/h of the structure with sapphire substrate ($\epsilon_\| = 11.6, \epsilon_\perp = 9.4$) [10] is presented in Fig. 3 with the parameters $\theta = 0, \pi/4$, and $\pi/2$. Fig. 3 shows that the capacitance for $\theta = \pi/2$ has larger value than one for $\theta = 0$. This fact is clear from (11), because α decreases from $\sqrt{\epsilon_\|/\epsilon_\perp}$ to $\sqrt{\epsilon_\perp/\epsilon_\|}$ as θ increases from 0 to $\pi/2$ in case of $\epsilon_\| > \epsilon_\perp$.

V. Conclusion

We analytically presented the capacitance and field distributions of the shielded-strip transmission line with the anisotropic medium. The concrete examples of two structures were also presented with the substrate of $\epsilon_\perp/\epsilon_\| = 1/9$ and the sapphire substrate.

The analytical procedure using the transformation of (3) and a conformal mapping technique is applicable to analysis of the shielded coupled stripline and the asymmetric stripline with an anisotropic medium.

Appendix

For K'/K, we obtain the following expansion in much the same as Riblet's procedure [11, appendix]:

$$\frac{K'}{K} = \frac{1}{\pi}\left\{2 \ln \frac{2k}{1-k'} - \frac{1}{4}\left(\frac{1-k'}{k}\right)^4 - \frac{13}{128}\left(\frac{1-k'}{k}\right)^8 - \frac{23}{384}\left(\frac{1-k'}{k}\right)^{12} - \frac{2701}{65536}\left(\frac{1-k'}{k}\right)^{16} - \cdots\right\}$$
$$\left(\text{for } k^2 \leqq 0.5\right). \tag{A-1}$$

Equation (A-1) was obtained by replacing k' in [11, eq. (II)] by

$$\left\{1-\left(\frac{1-k'}{k}\right)^2\right\}\Big/\left\{1+\left(\frac{1-k'}{k}\right)^2\right\}$$

and by expanding K'/K in [11, eq. (IV)]. The expansion of (A-1) is a variation of the first Hilberg's approximation [6, eq. (15)]. The convergence of (A-1) is superior to that of Riblet's expansion [11, eq. (VI)].

The expansion of K'/K for $k^2 > 0.5$ is obtained by replacing k, k', K, and K' in (A-1) by k', k, K', and K, respectively.

References

[1] W. Magnus and F. Oberhettinger, "Die berechnung des wellenwiderstandes," *Arch. Elektrotech.*, 37 Bund, heft 8, pp. 380–390, 1943.
[2] S. B. Cohn, "Characteristic impedance of the shielded-strip transmission line," *IRE Trans. Microwave Theory Tech.*, vol. MTT-2, pp. 52–57, July 1954.
[3] D. Park, "Planar transmission lines," *IRE Trans. Microwave Theory Tech.*, vol. MTT-3, pp. 8–12, Apr. 1955.
[4] R. E. Collin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1960, pp. 139–148.
[5] H. A. Wheeler, "Transmission-line properties of parallel strip separated by a dielectric sheet," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 172–185, Mar. 1965.
[6] W. Hilberg, "From approximation to exact relations for characteristic

impedance," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 259–265, May 1969.
[7] H. A. Wheeler, "Transmission-line properties of a strip line between parallel planes," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 866–876, Nov 1978.
[8] S. Kusase and R. Terakado, "Mapping theory of two-dimensional anisotropic regions," *Proc. IEEE*, vol. 67, Jan. 1979, pp. 171–172.
[9] F. Bowman, *Introduction to Elliptic Functions with Applications*. New York: Dover, 1961, pp. 71–72.
[10] J. Fontanella, C. Andeen, and D. Schuele, "Low-frequency dielectric constants of α-quartz, sapphire, MgF_2 and MgO," *J. Appl. Phys.*, vol. 45, pp. 2852–2854, July 1974.
[11] H. J. Riblet, "Two limiting values of the capacitance of symmetrical rectangular coaxial strip transmission line," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 661–666, July 1981.

Characteristics of Microstrip Directional Couplers on Anisotropic Substrates

NICOLAOS G. ALEXOPOULOS, SENIOR MEMBER, IEEE, AND S.A. MAAS

Abstract—**The properties of directional couplers fabricated on anisotropic substrates are examined on a quasistatic basis. Three substrate materials are considered: sapphire, Epsilam (a ceramic-filled polytetrafluoroethylene dielectric), and boron nitride. VSWR, directivity, and coupling are presented for several representative 10-dB coupler designs, as well as variations in these parameters with crystal axis offsets. It is demonstrated that high directivity can be achieved by making use of the substrate anisotropy in conjunction with a top cover to equalize even- and odd-mode phase velocities.**

I. Introduction

The need to improve the performance repeatability of microwave hybrid integrated circuits has generated considerable interest in the use of crystalline substrate materials such as sapphire and boron nitride with highly predictable dielectric properties. At the same time, the need for low cost microstrip circuits has generated a number of glass- and ceramic-filled polymeric materials, by such trade names as Duroid and Epsilam. These materials are electrically anisotropic either because of their crystalline structure or the processes used in their manufacture. In fact, it has been demonstrated that other, presumably isotropic materials such as alumina, show significant process-dependent anisotropy.

Anisotropy in microstrip substrates has traditionally been regarded as an undesirable property, primarily because microstrip impedances and phase velocities are somewhat more difficult to determine for anisotropic than for isotropic materials. However, in the case of directional couplers and other coupled-line components, certain types of anisotropy can be of significant advantage. The odd-mode phase velocity in a microstrip coupler using an isotropic substrate is greater than the even-mode velocity causing degradation of the coupler's directivity. This degradation becomes worse as the coupling is decreased or as the dielectric permittivity is increased. Furthermore, the effects of unmatched phase velocities are more severe than the effects of incorrect even- and odd-mode impedances. For example, a 10 percent difference in phase velocities reduces the directivity of a 10-dB coupler to 13 dB from its theoretically infinite value with equal phase velocities. On the other hand, a 10 percent error in either even- or odd-mode impedance, with equal phase velocities, reduces the directivity only to 26 dB. The corresponding figures for a 20-dB coupler are 2 dB and 26 dB, respectively. Invariably, the even-mode phase velocity is less than the odd-mode velocity by 10–12 percent for a 10-dB coupler, depending on the permittivity of the dielectric substrate.

Manuscript received December 29, 1981; revised March 2, 1982. This work was supported in part by U.S. Army Contract DAAG-29-79-C-0050.

N. G. Alexopoulos is with the Electrical Engineering Department, University of California, Los Angeles, CA 90024.
S. A. Maas is with TRW, Redondo Beach, CA 90278.

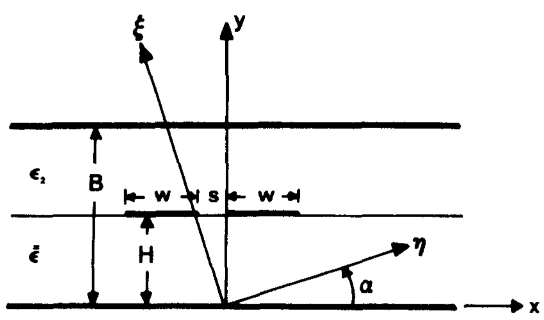

Fig. 1. Microstrip geometry showing crystal (ξ, η) and microstrip (x, y) axis.

However, if the substrate is anisotropic and the permittivity in a direction parallel to the ground plane is greater than the perpendicular component [1], the odd-mode phase velocity will be reduced relative to the even mode and significantly improved directivity will result [2]. To equalize the phase velocities, the parallel component (the $x-z$ component of Fig. 1) must be approximately twice the perpendicular y-component [1],[2]. This degree of anisotropy is virtually nonexistent in practical microwave substrate materials, although a factor of 1.2–1.5 has been observed. Nevertheless, these practically achievable anisotropies can be used to significantly improve coupler directivity, and changes in the microstrip geometry—most notably in the top cover height for shielded microstrip—can be used in conjunction with anisotropy to equalize even- and odd-mode phase velocities.

II. Analysis

Anisotropic materials have received recently considerable attention [3]–[7] as microstrip substrates. In this article, the anisotropy is described by a tensor relative permittivity given by

$$\bar{\bar{\epsilon}} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{yx} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix}. \tag{1}$$

If the axes of the substrate are aligned with the axes of the crystal (see Fig. 1) then $\epsilon_{yx} = \epsilon_{xy} = 0$. If, however, the alignment is imperfect, the elements of $\bar{\bar{\epsilon}}$ above are given by

$$\epsilon_{xx} = \epsilon_{\xi\xi} \cos^2\alpha + \epsilon_{\eta\eta} \sin^2\alpha \tag{2}$$

$$\epsilon_{yy} = \epsilon_{\xi\xi} \sin^2\alpha + \epsilon_{\eta\eta} \cos^2\alpha \tag{3}$$

and

$$\epsilon_{xy} = \epsilon_{yx} = (\epsilon_{\xi\xi} - \epsilon_{\eta\eta}) \sin\alpha \cos\alpha \tag{4}$$

where the ξ, η subscripts refer to the crystal axes. As discussed in [8], a quasi-static analysis of the problem is sufficient for lower frequencies. In order to compute the characteristic impedance and phase velocity of the coupled lines, an even–odd mode

0018-9480/82/0800-1267$00.75 ©1982 IEEE